Highly excited and exotic meson spectroscopy from lattice QCD

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With Jo Dudek, Robert Edwards, Mike Peardon,
David Richards and the *Hadron Spectrum Collaboration*

Overview – reminder

Light meson spectroscopy

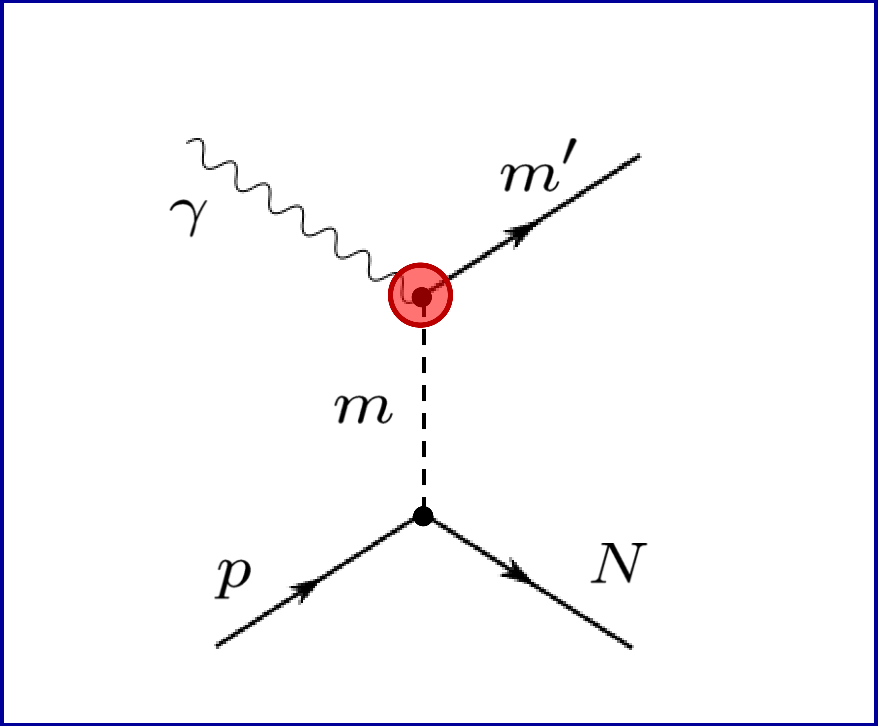
GlueX (JLab), BESIII, PANDA

Exotics (**1** $^{-+}$, ...)?

Photocouplings

Extracting excited meson spectra using Lattice QCD...

Photoproduction at GlueX (JLab 12 GeV upgrade)



Spectroscopy on the lattice

Calculate energies and matrix elements (“overlaps”, Z) from correlation functions of meson interpolating fields

$$O(t) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \; \bar{\psi}(x)\Gamma_i \overleftrightarrow{D}_j \overleftrightarrow{D}_k \ldots \psi(x)$$

$$C_{ij}(t) \;\; = \;\; < 0|O_i(t)O_j(0)|0 >$$

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$$\begin{aligned} C_{ij}(t) &= < 0|O_i(t)O_j(0)|0 > \\ &= \sum_n \frac{e^{-\boxed{E_n}t}}{2\boxed{E_n}} < 0|O_i(0)|n >< n|O_j(0)|0 > \end{aligned}$$

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Variational Method

Large basis of operators → matrix of correlators $C_{ij}(t)$ (N x N matrix)

Generalised eigenvector problem:

$$C_{ij}(t)v_j^{(n)} = \lambda^{(n)}(t)C_{ij}(t_0)v_j^{(n)}$$

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$$\lambda^{(n)}(t) \rightarrow e^{-E_n(t-t_0)}$$

($t >> t_0$)

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Eigenvectors → optimal linear combination of operators to overlap on to a state

$$\Omega^{(n)} \sim \sum_i v_i^{(n)} O_i$$

$Z^{(n)}$ related to eigenvectors

$$Z_i^{(n)} \equiv < 0|O_i|n >$$

Spin on the lattice

On a lattice, 3D rotation group is broken to Octahedral Group

3D in continuum:

Infinite number of *irreps*: J = 0, 1, 2, 3, 4, ...

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<u>3D in continuum:</u>

Infinite number of *irreps*: J = 0, 1, 2, 3, 4, ...

<u>On lattice:</u>

Finite number of *irreps*: A_1, A_2, T_1, T_2, E

Irrep	A_1	A_2	T_1	**T_2**	**E**
dim	1	1	3	3	2
cont. spins	0,4,6, ...	3,6,7, ...	1,3,4, ...	**2**,3,4, ...	**2**,4,5, ...

(and others for half-integer spin)

Spin and operator construction

Construct operators which only overlap on to one spin in the continuum limit

$$\langle 0|\mathcal{O}^{J,M}|J', M'\rangle = Z^{[J]}\delta_{J,J'}\delta_{M,M'}$$

'Subduce' operators on to lattice irreps (J → Λ):

$$\langle 0|\mathcal{O}^{[J]}_{\Lambda,\lambda}|J', M\rangle = \mathcal{S}^{J,M}_{\Lambda,\lambda} Z^{[J]}\delta_{J,J'}$$

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- As an example: three degenerate 'light' quarks ($N_f = 3$, $M_\pi \approx 700$ MeV)
- Dynamical (unquenched). Only connected diagrams (isovectors and kaons)

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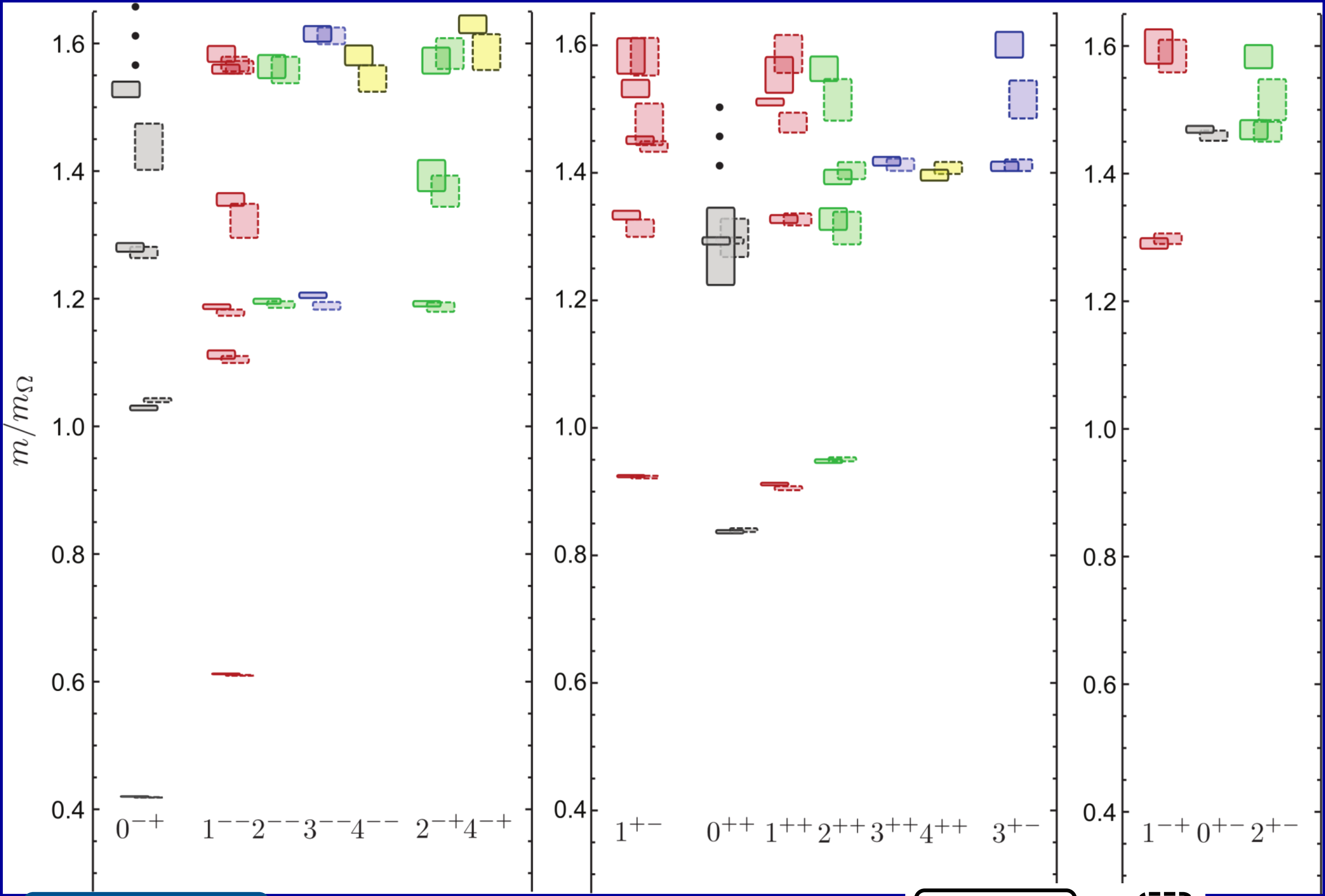
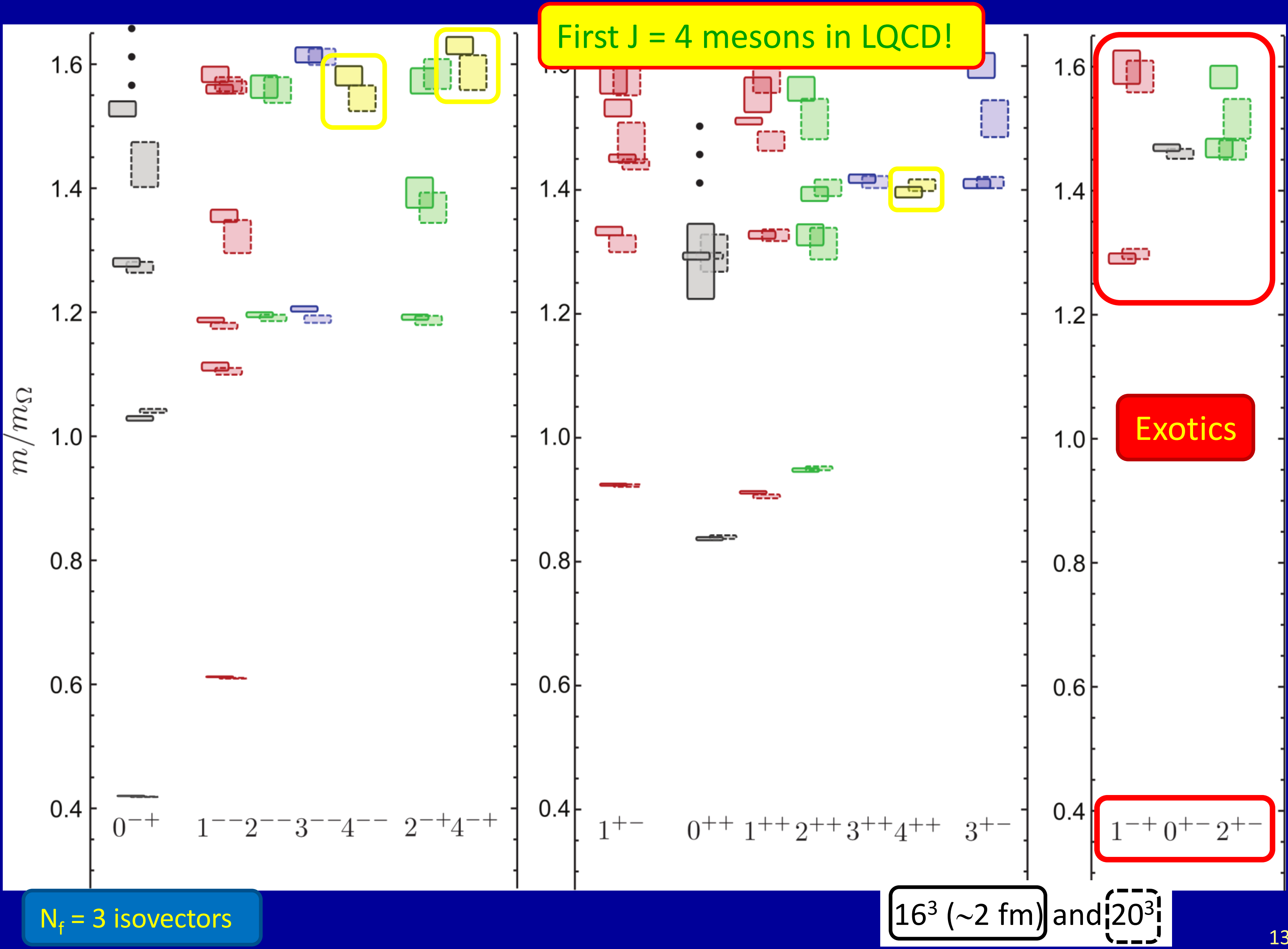


$N_f = 3$ isovectors

16^3 (~2 fm) and 20^3



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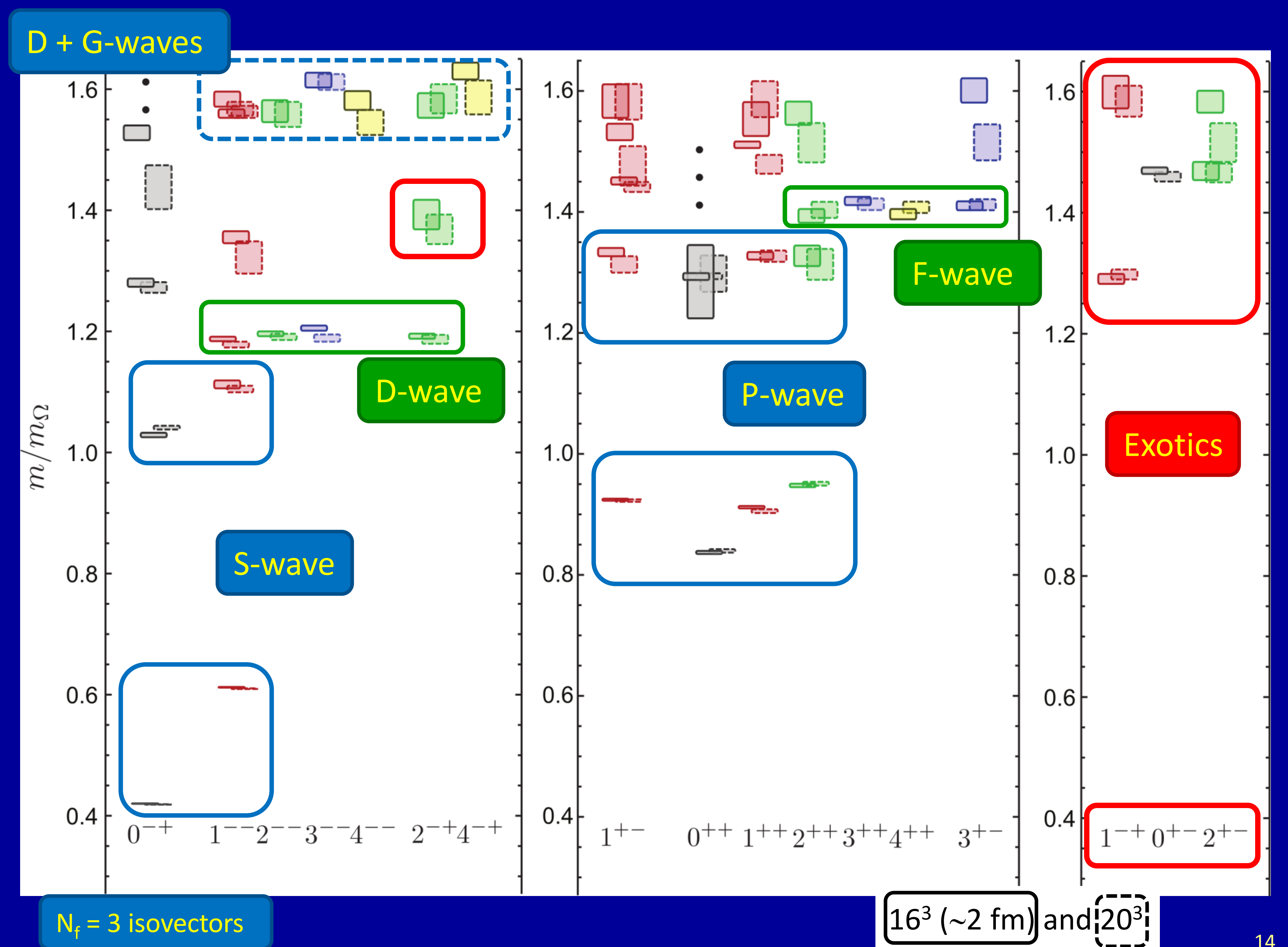

D + G-waves
D-wave
S-wave
P-wave
F-wave
Exotics
m/m_Ω
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0^{-+} 1^{--} 2^{--} 3^{--} 4^{--} 2^{-+} 4^{-+}
1^{+-} 0^{++} 1^{++} 2^{++} 3^{++} 4^{++} 3^{+-}
1^{-+} 0^{+-} 2^{+-}
$N_f = 3$ isovectors
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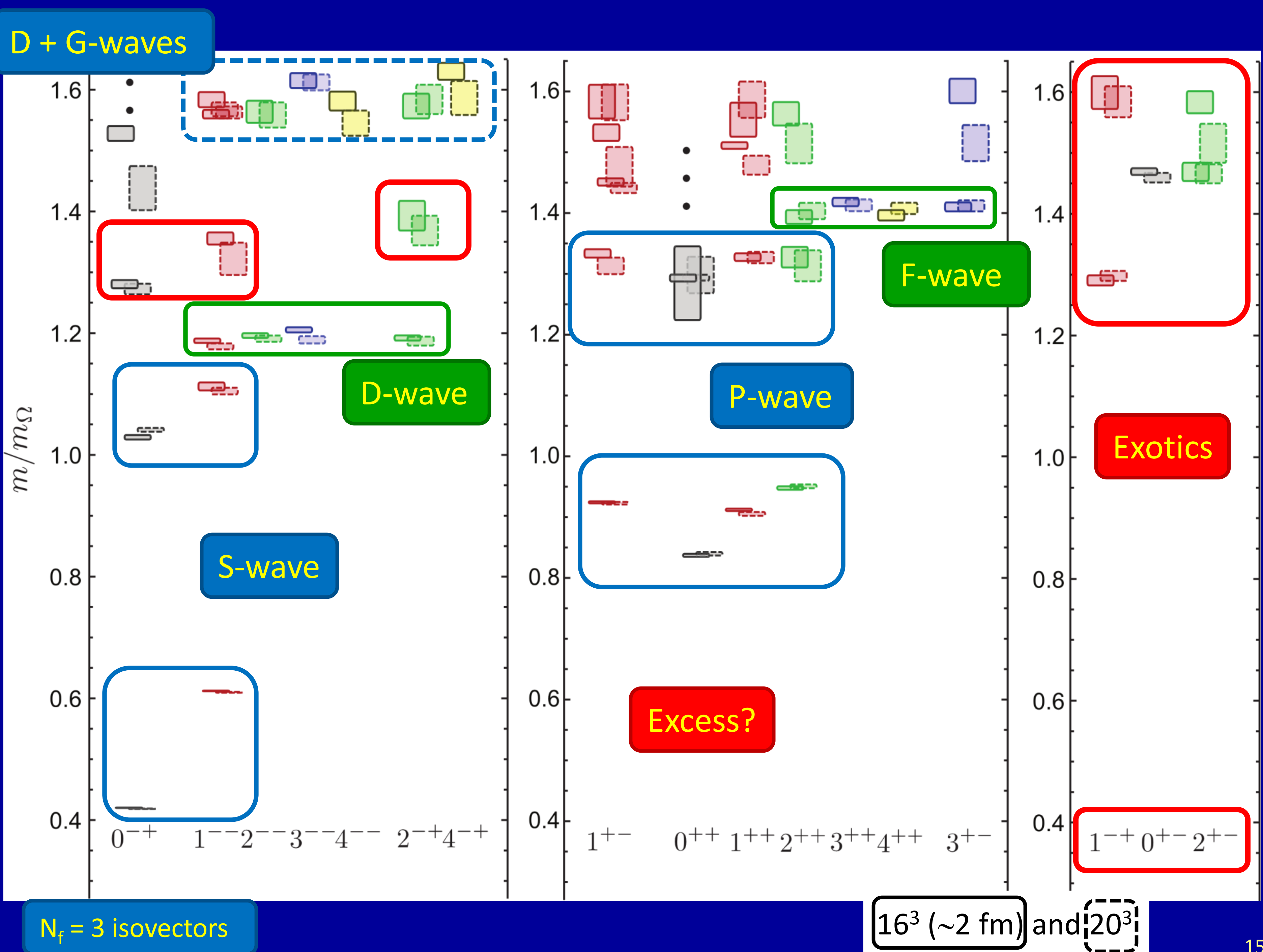
D + G-waves
1.6
1.4
1.2
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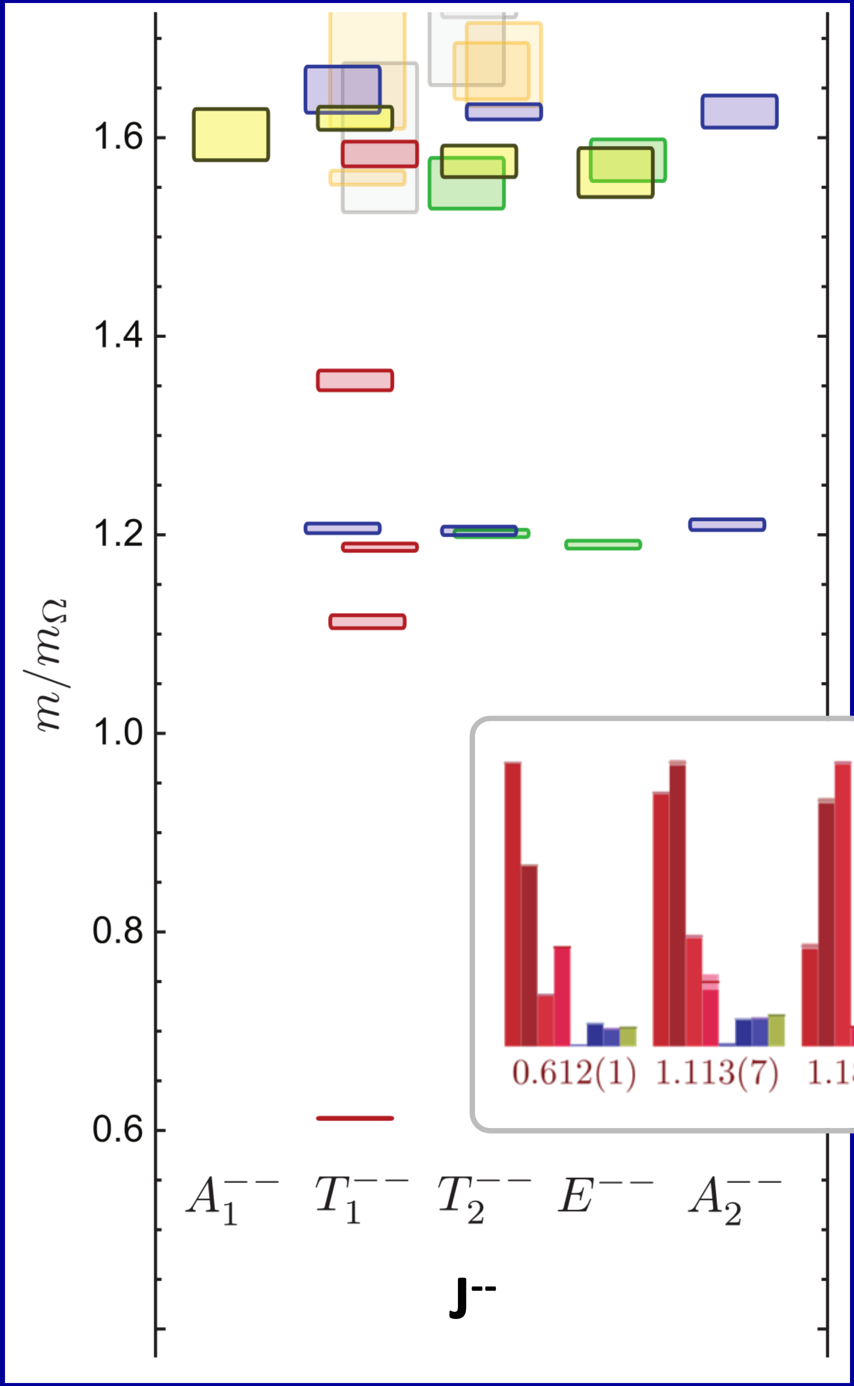
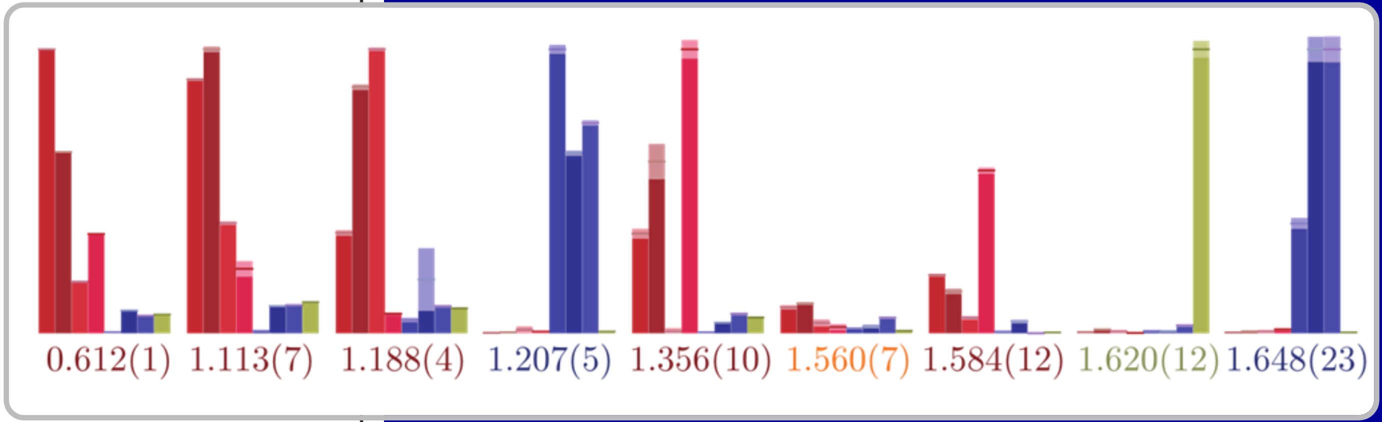
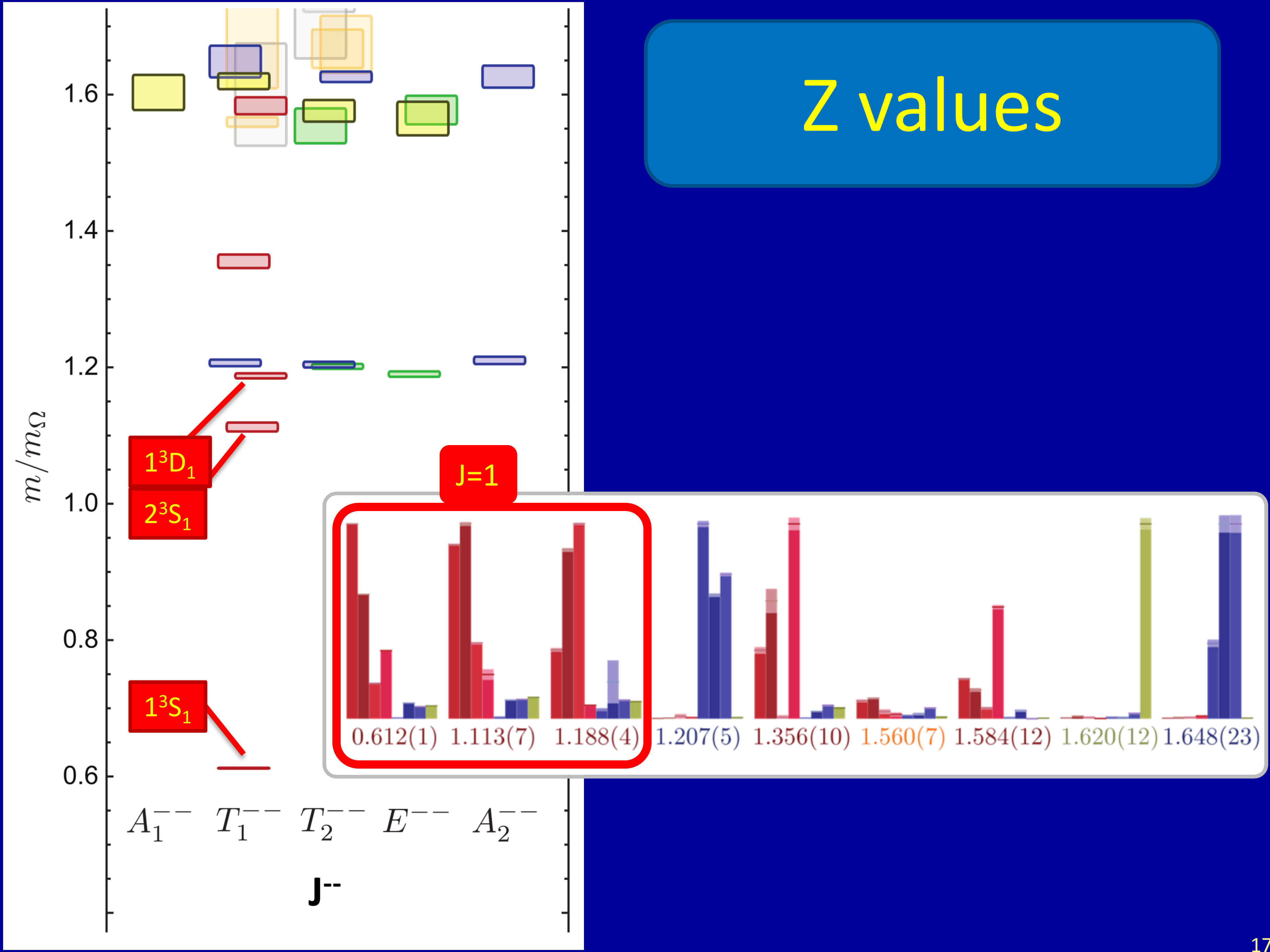
0.8
0.6
0.4
m/m_Ω
D-wave
S-wave
0^{-+} 1^{--} 2^{--} 3^{--} 4^{--} 2^{-+} 4^{-+}
F-wave
P-wave
Excess?
1^{+-} 0^{++} 1^{++} 2^{++} 3^{++} 4^{++} 3^{+-}
Exotics
1^{-+} 0^{+-} 2^{+-}
N_f = 3 isovectors
16^3 (~2 fm) and 20^3

Z values


1.6
1.4
1.2
1.0
0.8
0.6
m/m_Ω
A_1^{--} T_1^{--} T_2^{--} E^{--} A_2^{--}
J--

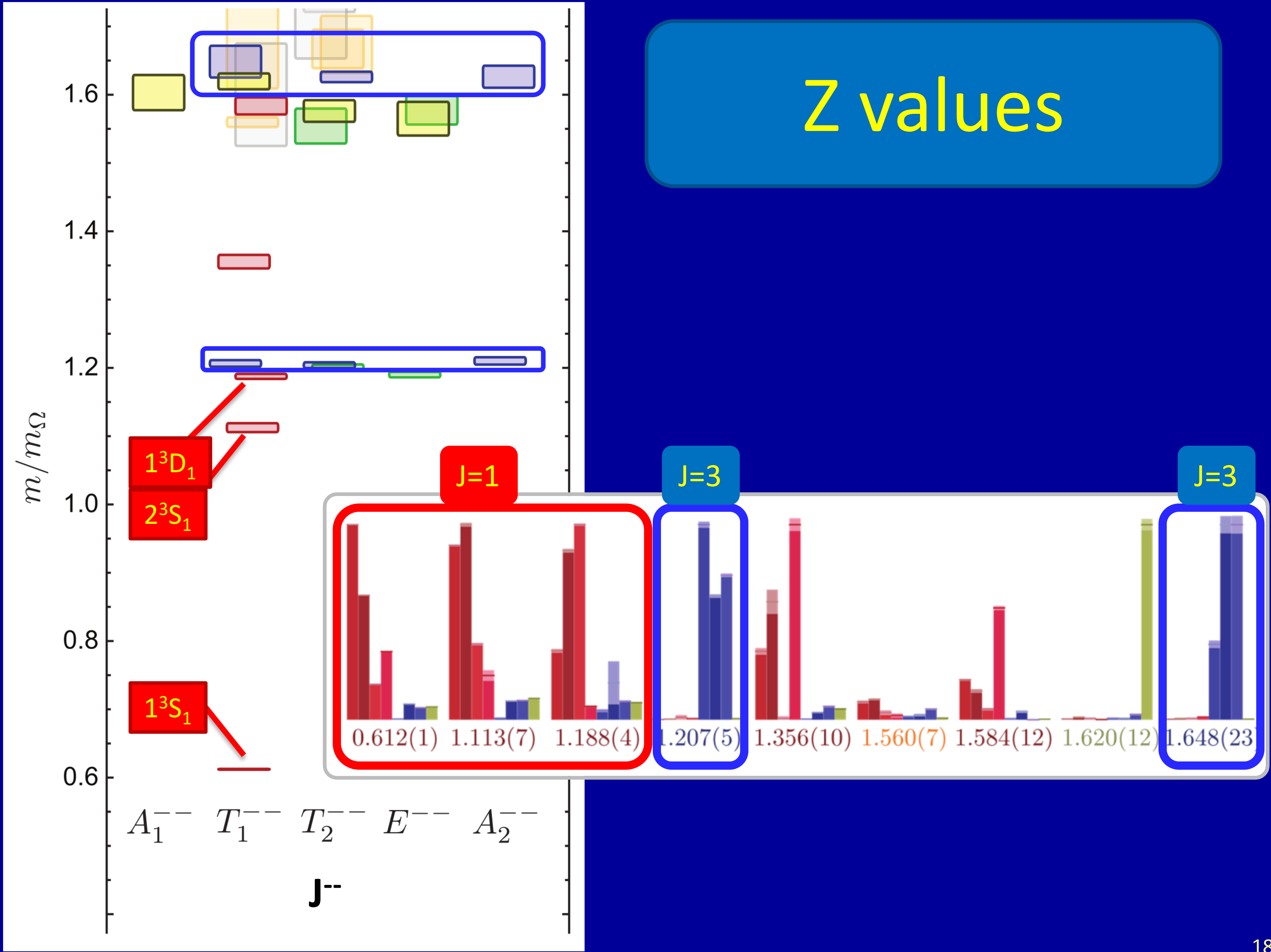

0.612(1)
1.113(7)
1.188(4)
1.207(5)
1.356(10)
1.560(7)
1.584(12)
1.620(12)
1.648(23)

Z values



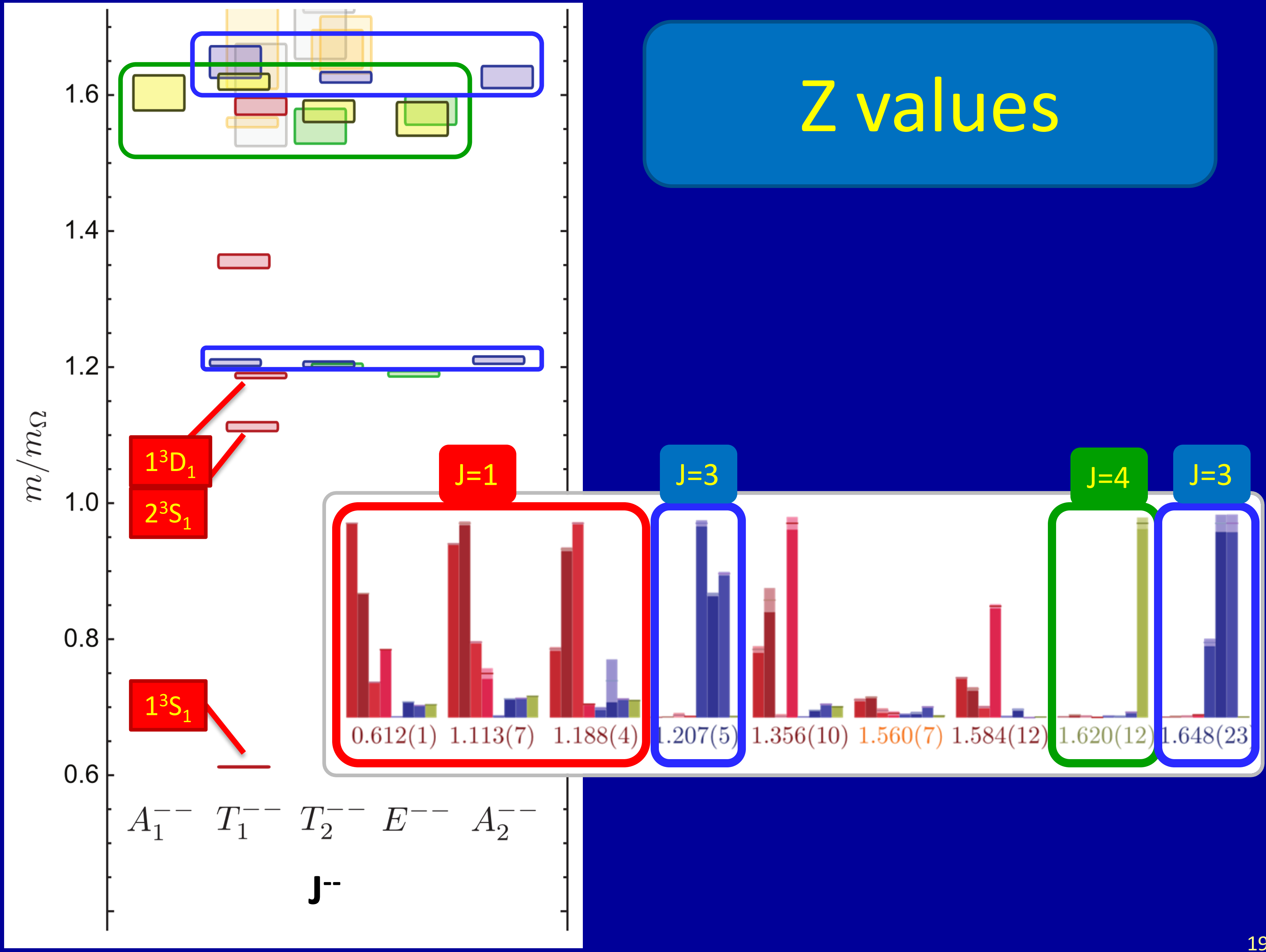
1.6
1.4
1.2
1.0
0.8
0.6
m/m_Ω
1^3D_1
2^3S_1
1^3S_1
A_1^{--} T_1^{--} T_2^{--} E^{--} A_2^{--}
J--
J=1
0.612(1) 1.113(7) 1.188(4) 1.207(5) 1.356(10) 1.560(7) 1.584(12) 1.620(12) 1.648(23)

Z values



1.6
1.4
1.2
1.0
0.8
0.6
m/m_Ω
1^3D_1
2^3S_1
1^3S_1
A_1^{--} T_1^{--} T_2^{--} E^{--} A_2^{--}
J--
J=1
J=3
J=3
0.612(1) 1.113(7) 1.188(4) 1.207(5) 1.356(10) 1.560(7) 1.584(12) 1.620(12) 1.648(23)

Z values



1.6
1.4
1.2
1.0
0.8
0.6
m/m_Ω
1^3D_1
2^3S_1
1^3S_1
A_1^{--} T_1^{--} T_2^{--} E^{--} A_2^{--}
J--
J=1
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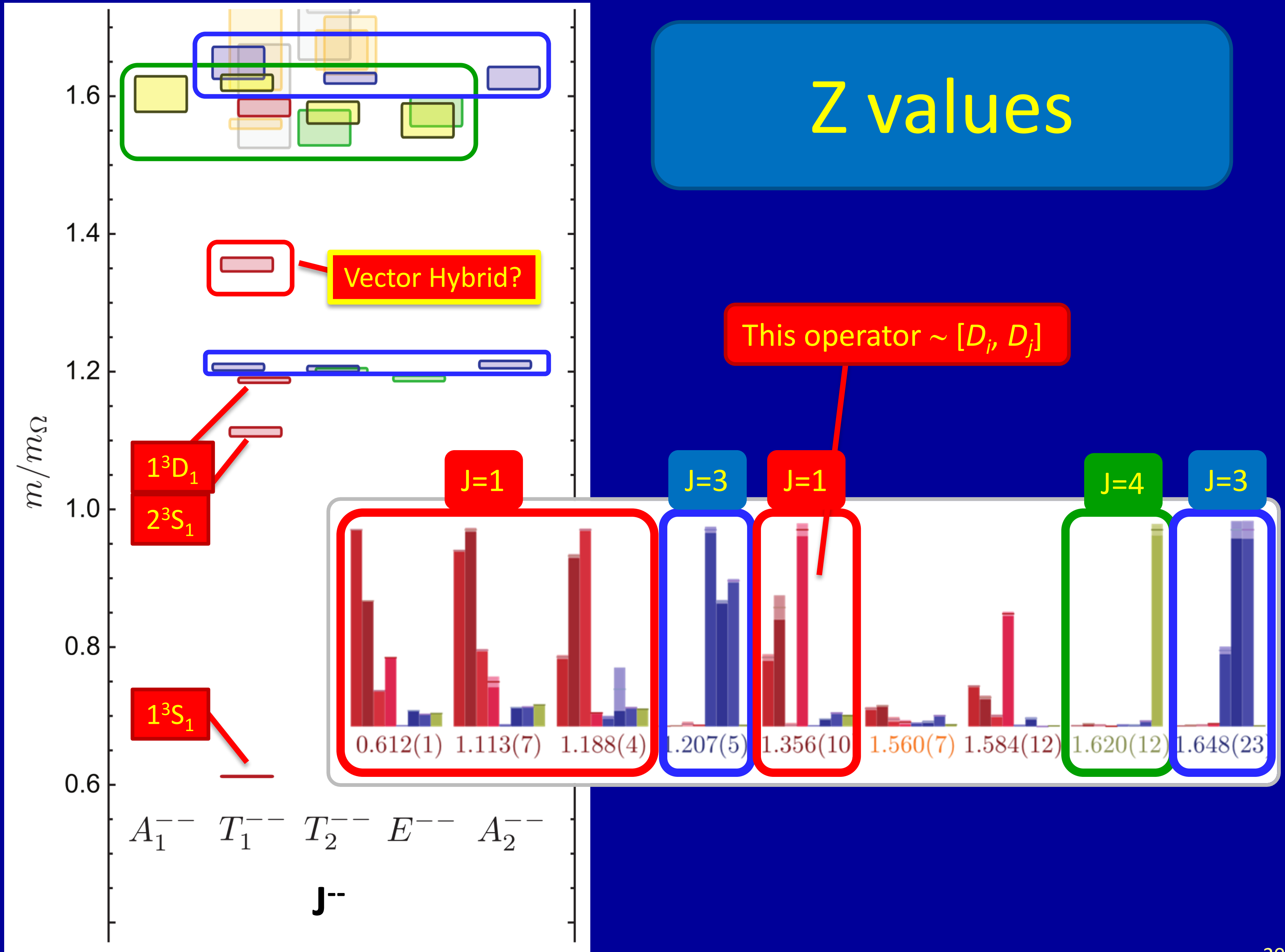
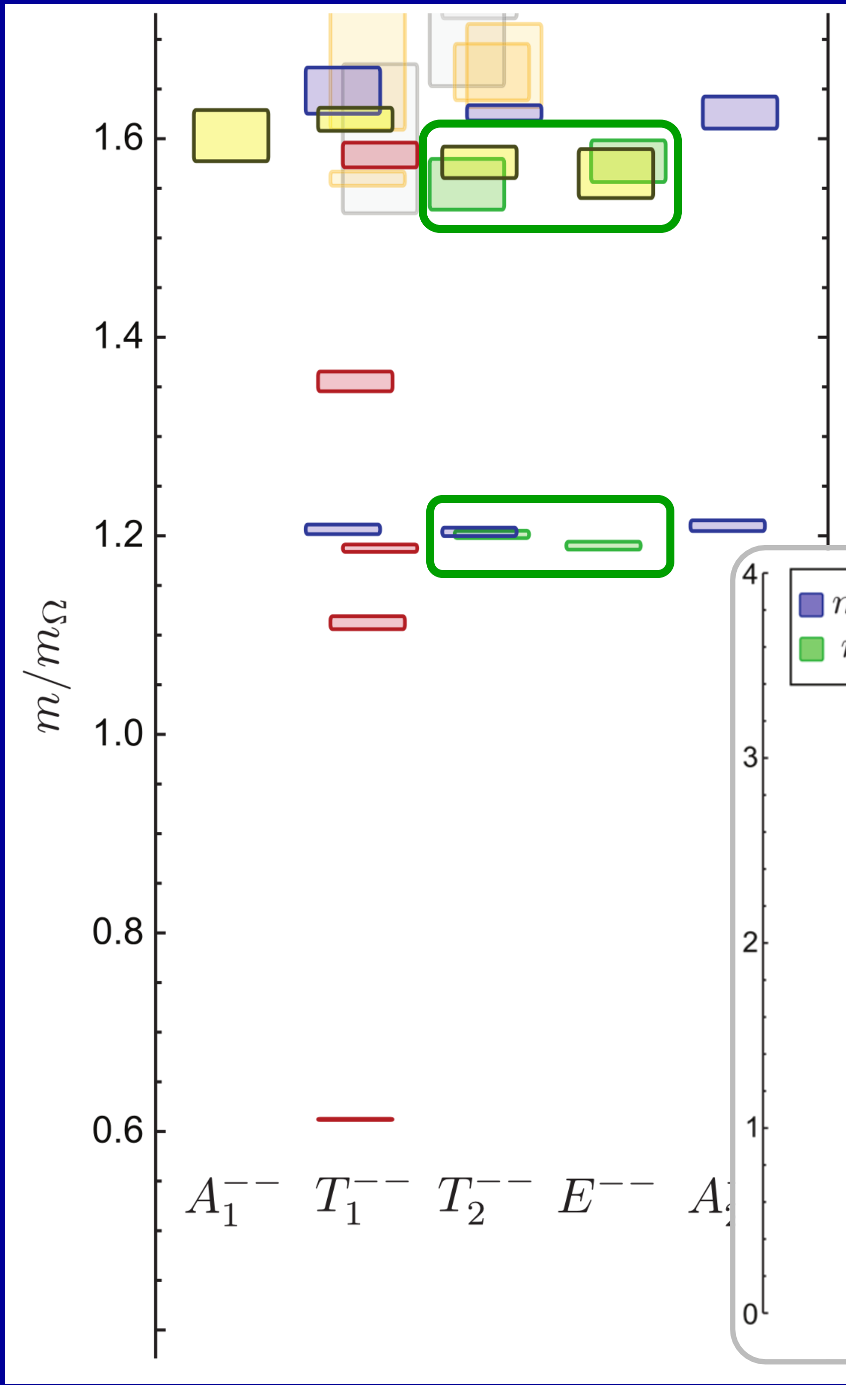
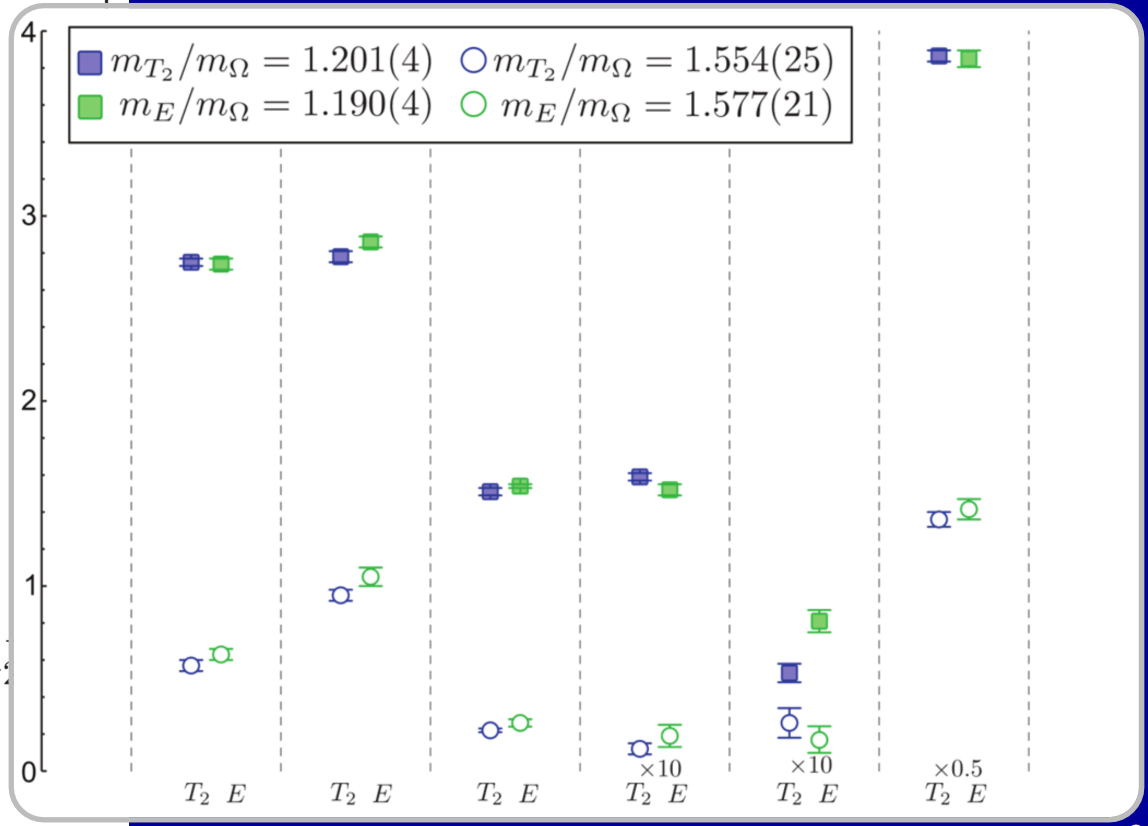
0.612(1) 1.113(7) 1.188(4) 1.207(5) 1.356(10) 1.560(7) 1.584(12) 1.620(12) 1.648(23)

Z values

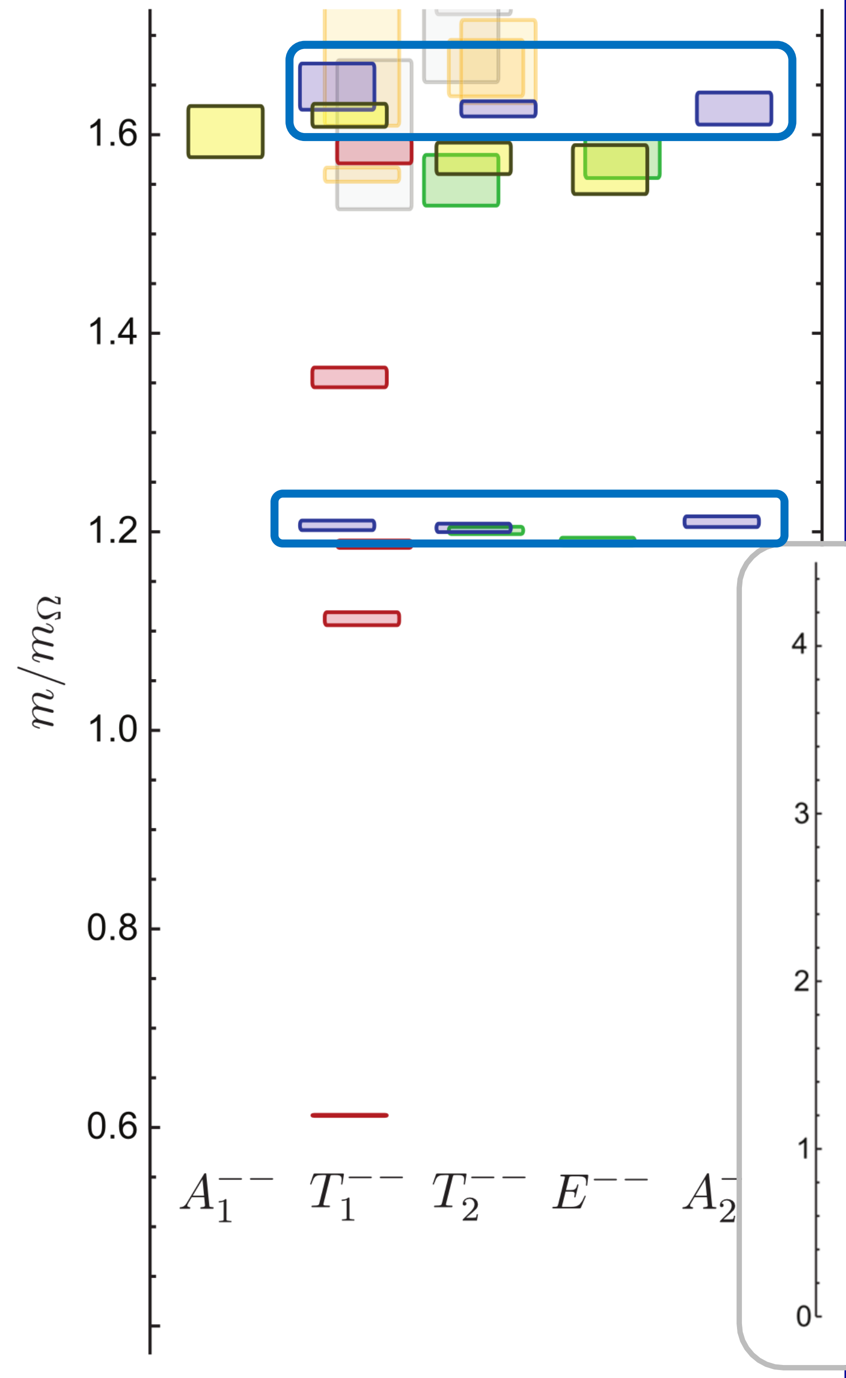

1.6
1.4
1.2
1.0
0.8
0.6
m/m_Ω
A_1^{--} T_1^{--} T_2^{--} E^{--} A_2^{--}
J--
Vector Hybrid?
1^3D_1
2^3S_1
1^3S_1
This operator ~ $[D_i, D_j]$
J=1
J=3
J=1
J=4
J=3
0.612(1) 1.113(7) 1.188(4) 1.207(5) 1.356(10 1.560(7) 1.584(12) 1.620(12) 1.648(23

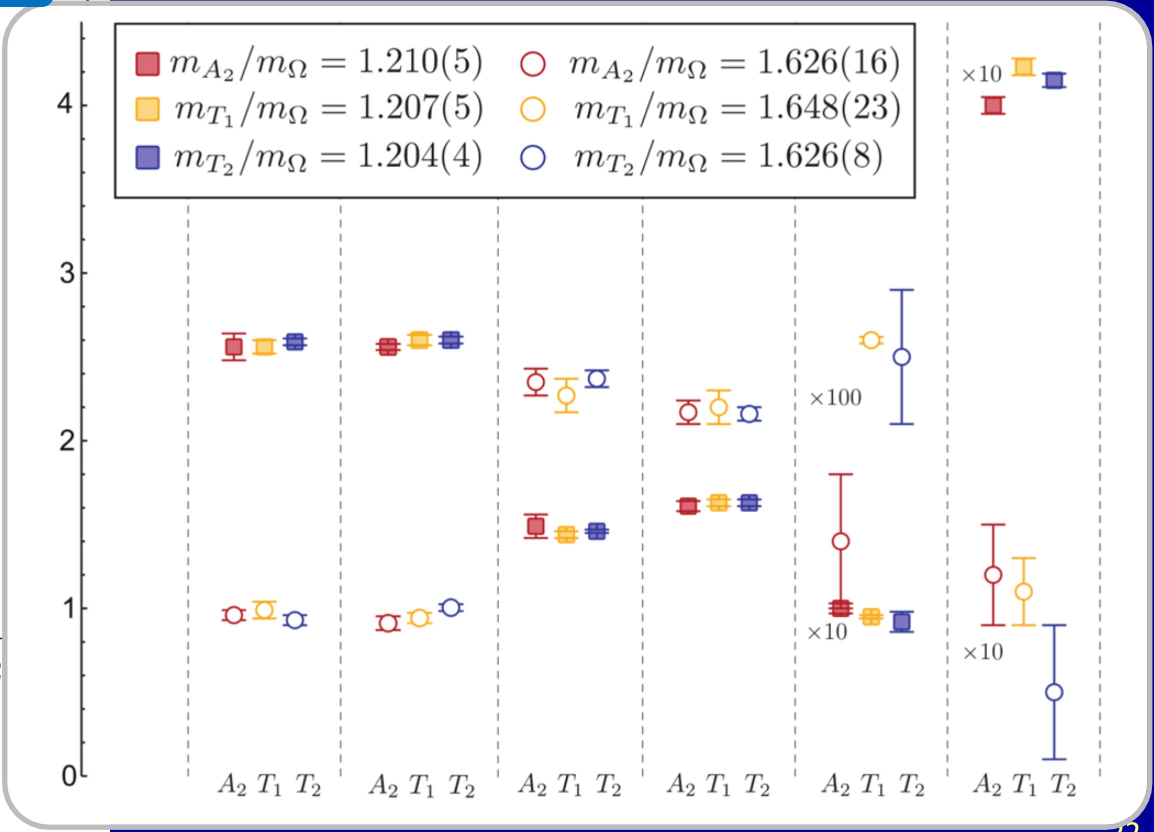
Z values – spin 2



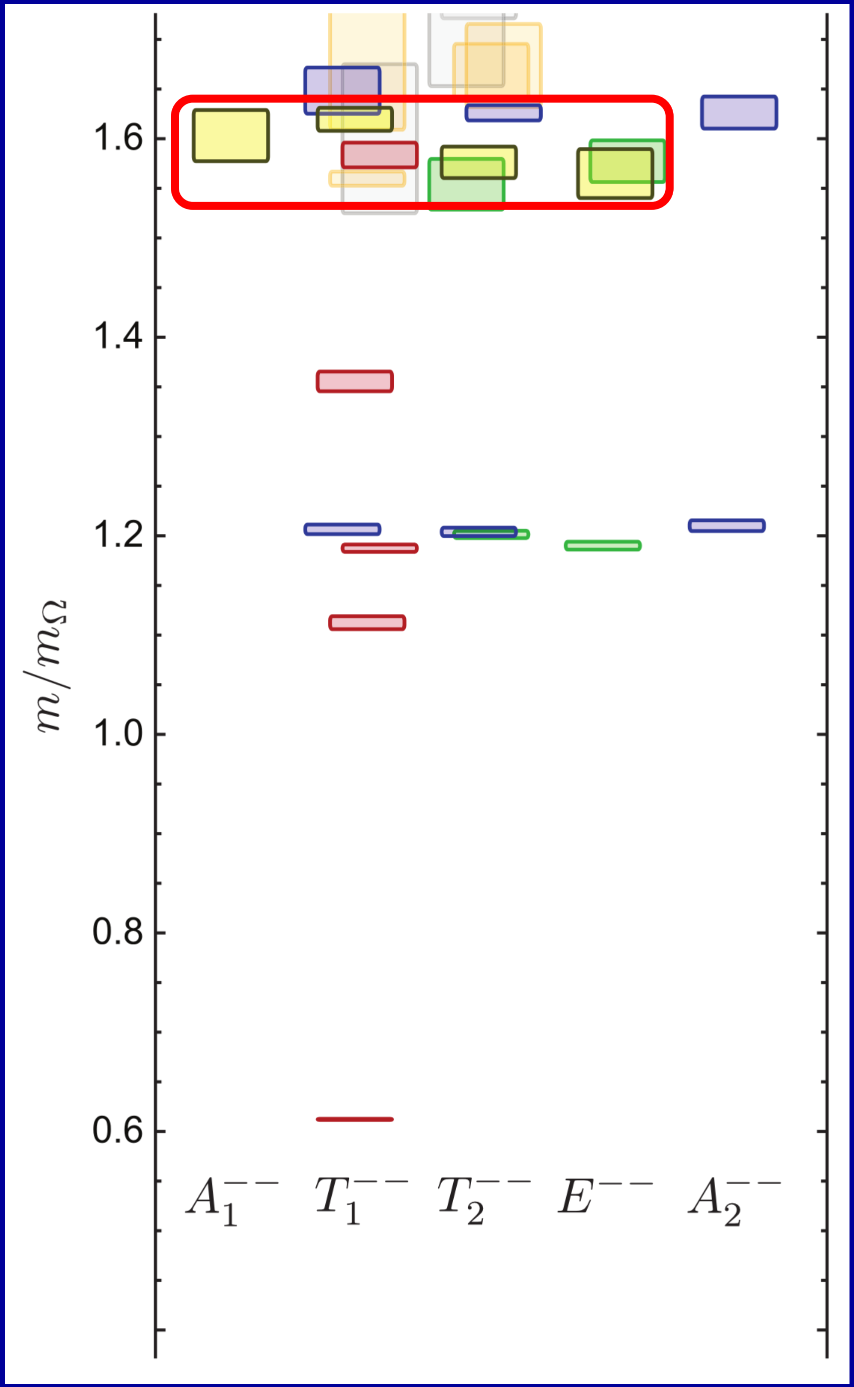


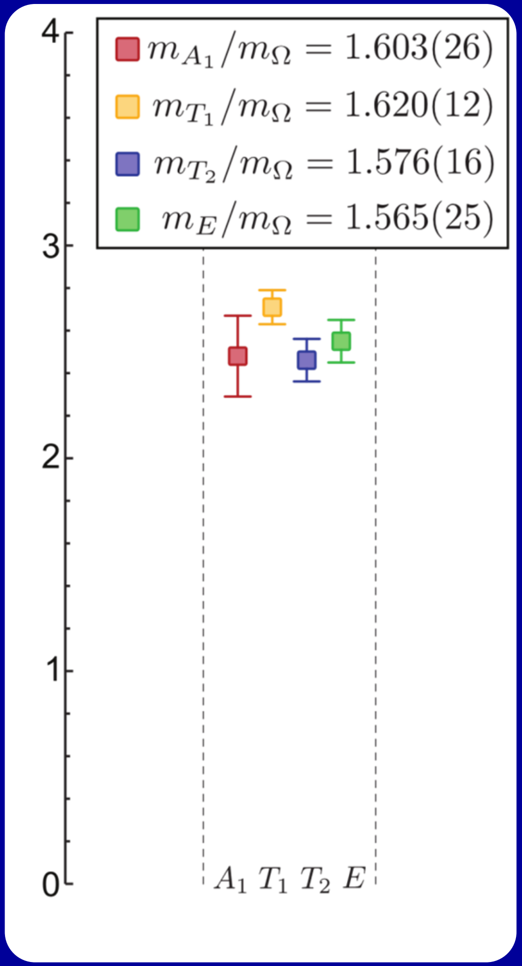
Z values – spin 3

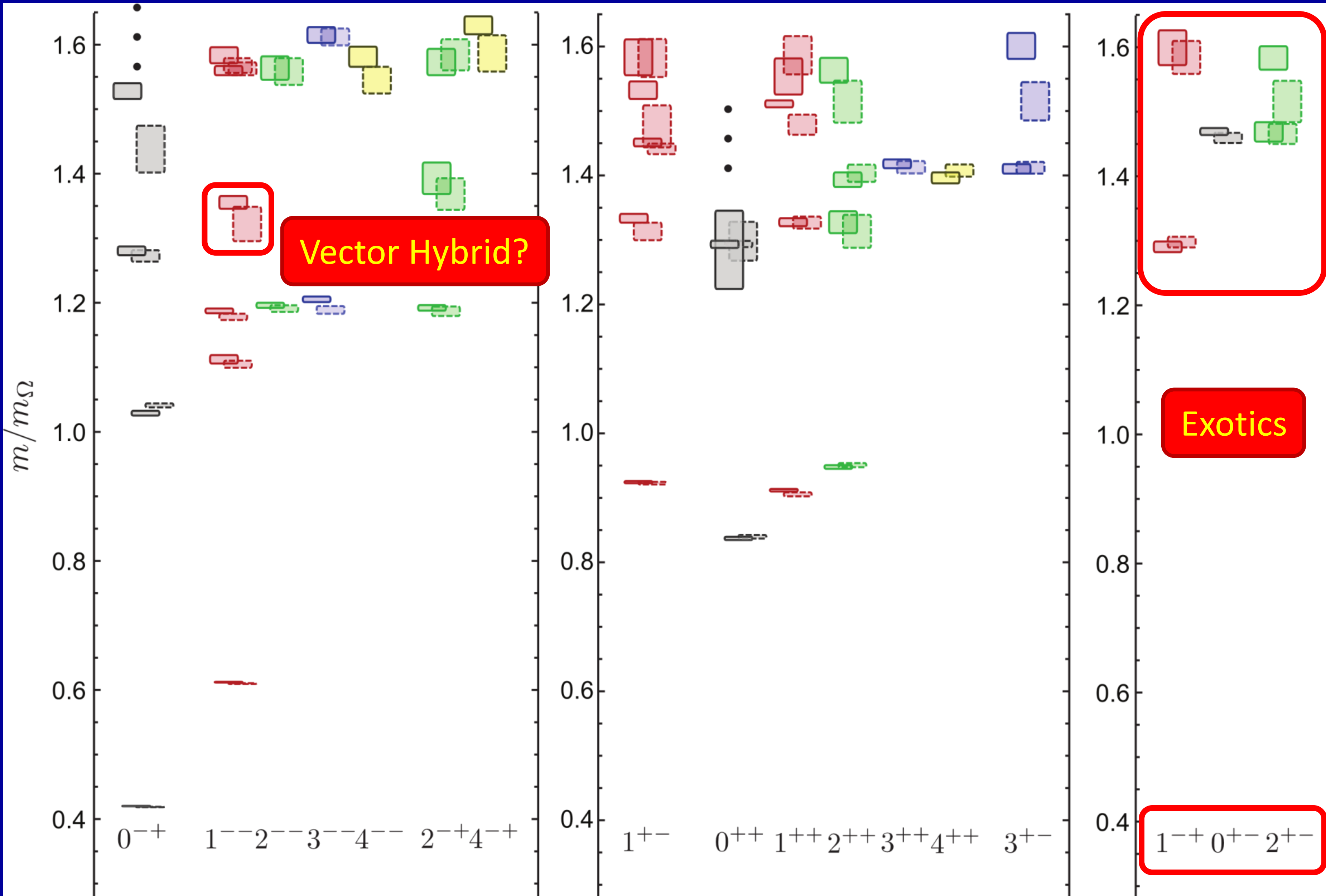




Z values – spin 4

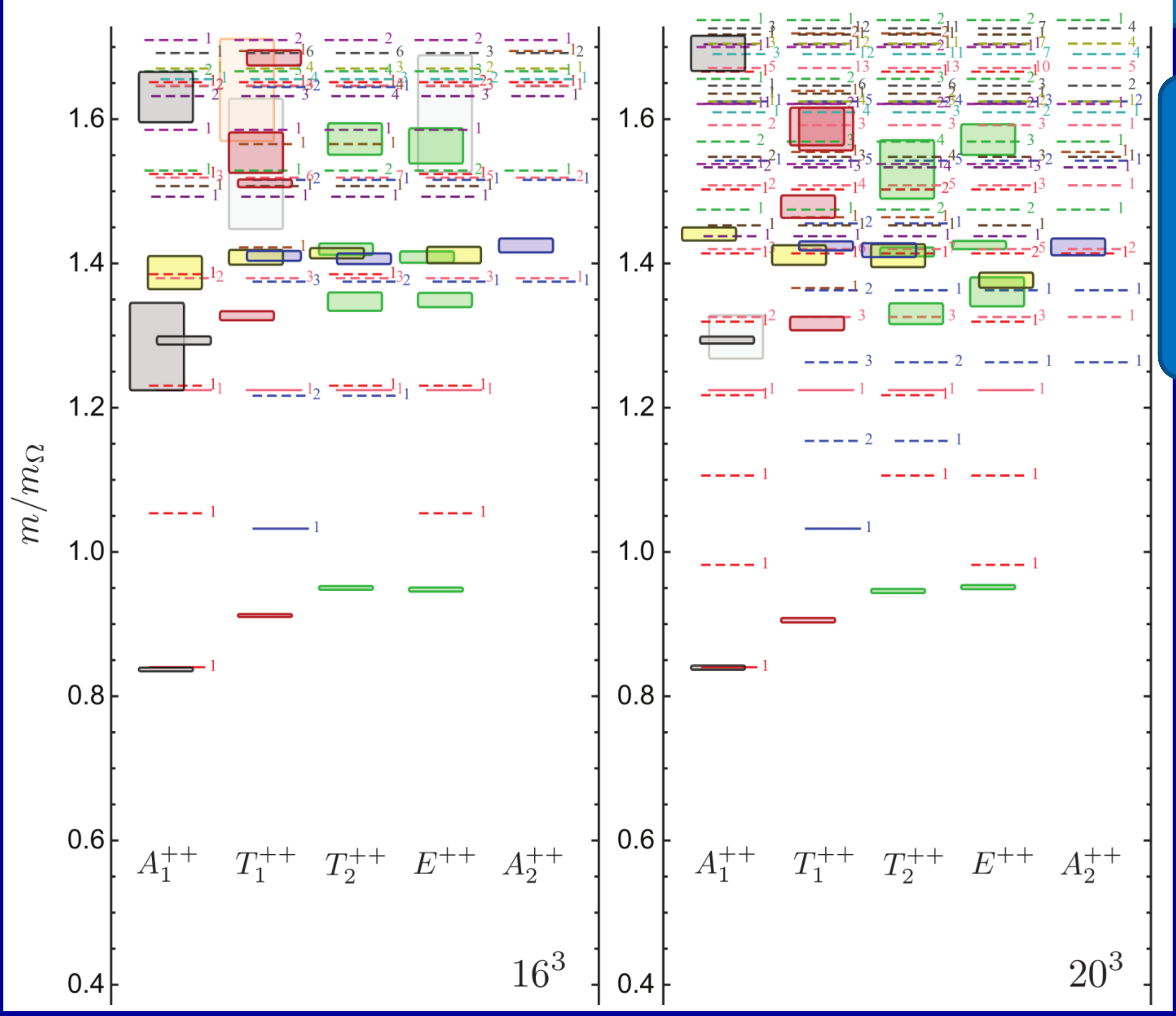






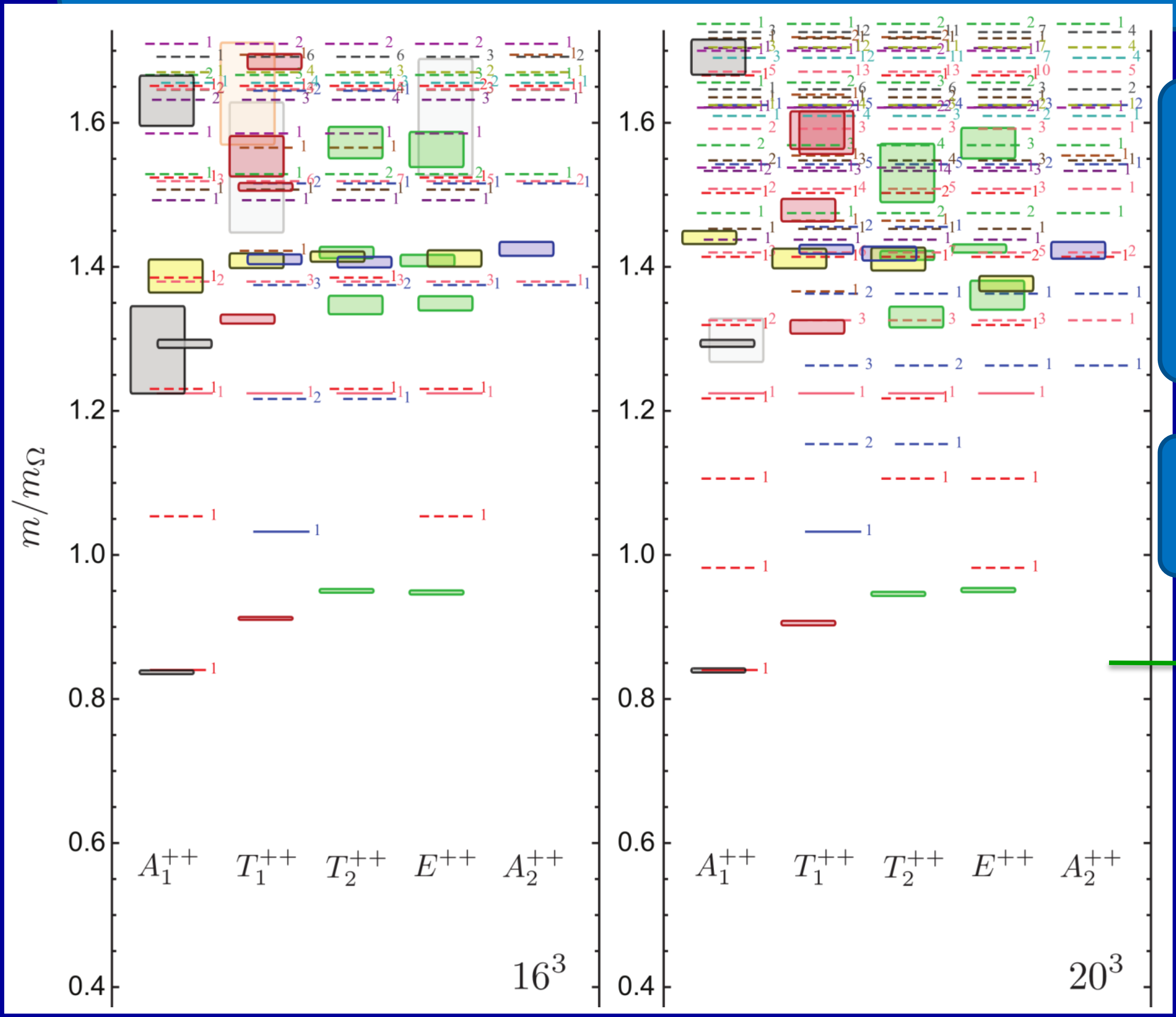
N_f = 3 isovectors

Multi-particle states?



On lattice: discrete allowed momenta – discrete spectrum of multiparticle states

Multi-particle states?



On lattice: discrete allowed momenta – discrete spectrum of multiparticle states

Expect two-meson states above $2m_\pi$

$2m_\pi \sim 0.85\ m_\Omega$

Where are they?

Summary and Outlook

Summary

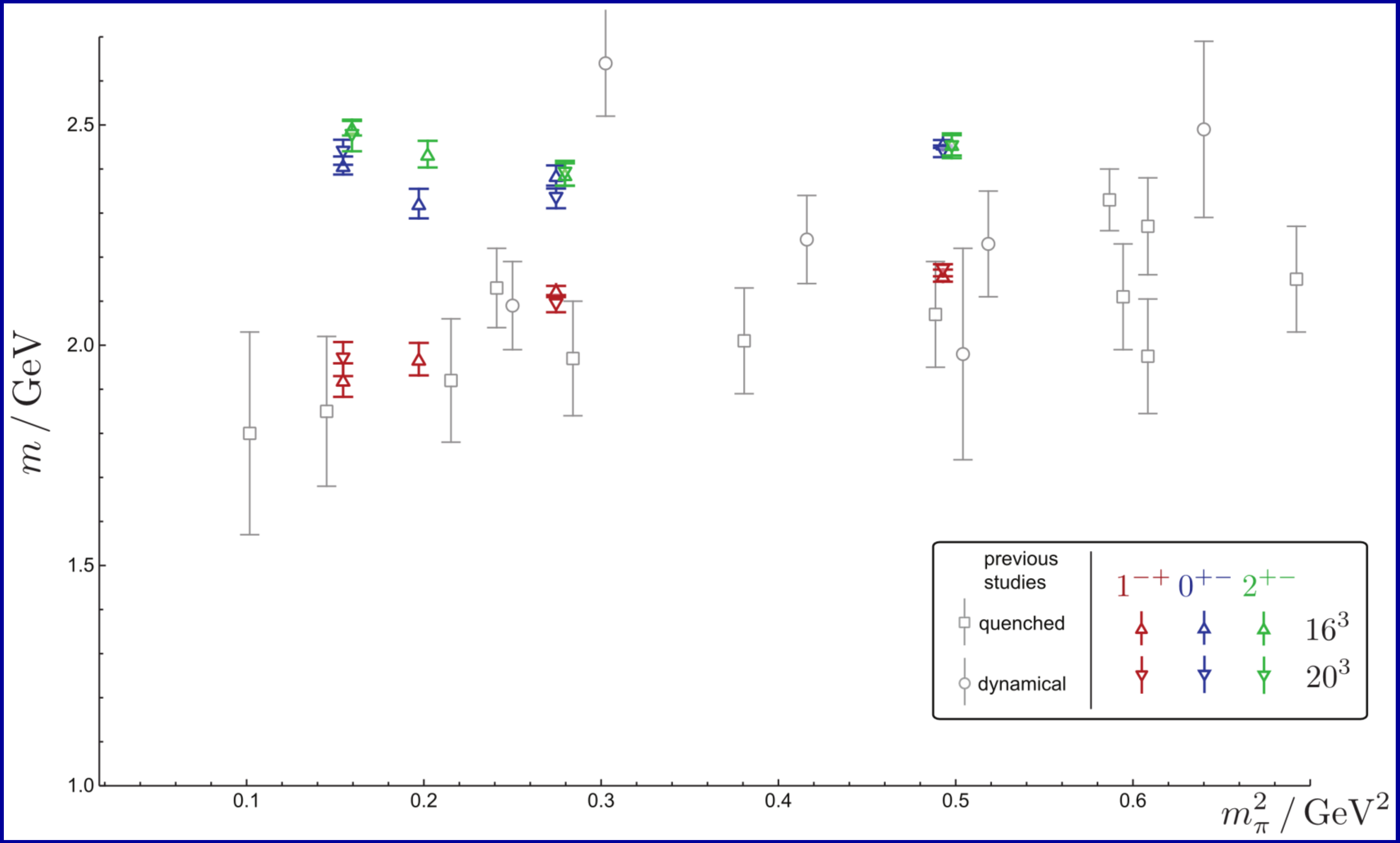
- First results on light mesons – technology and method work
- **Spin identification** is possible using operator overlaps
- **First spin 4** meson extracted and confidently identified on lattice
- **Exotics** (and non-exotic **hybrids**?)

Outlook – ongoing work

- Multi-meson operators
- Disconnected diagrams – isoscalars
- Baryons (Robert Edwards’ talk on Friday)
- Photocouplings

Extra Slides

Exotics summary



Kaons

Lower the light quark mass ($N_f = 2+1$) — SU(3) sym breaking

M_π / MeV	700	520	440	400
M_K / M_π	1	1.2	1.3	1.4

c.f. physical
$M_K / M_\pi = 3.5$

No longer have (a generalisation) of C-parity as a good quantum number

Combine J^{P+} and J^{P-} operators

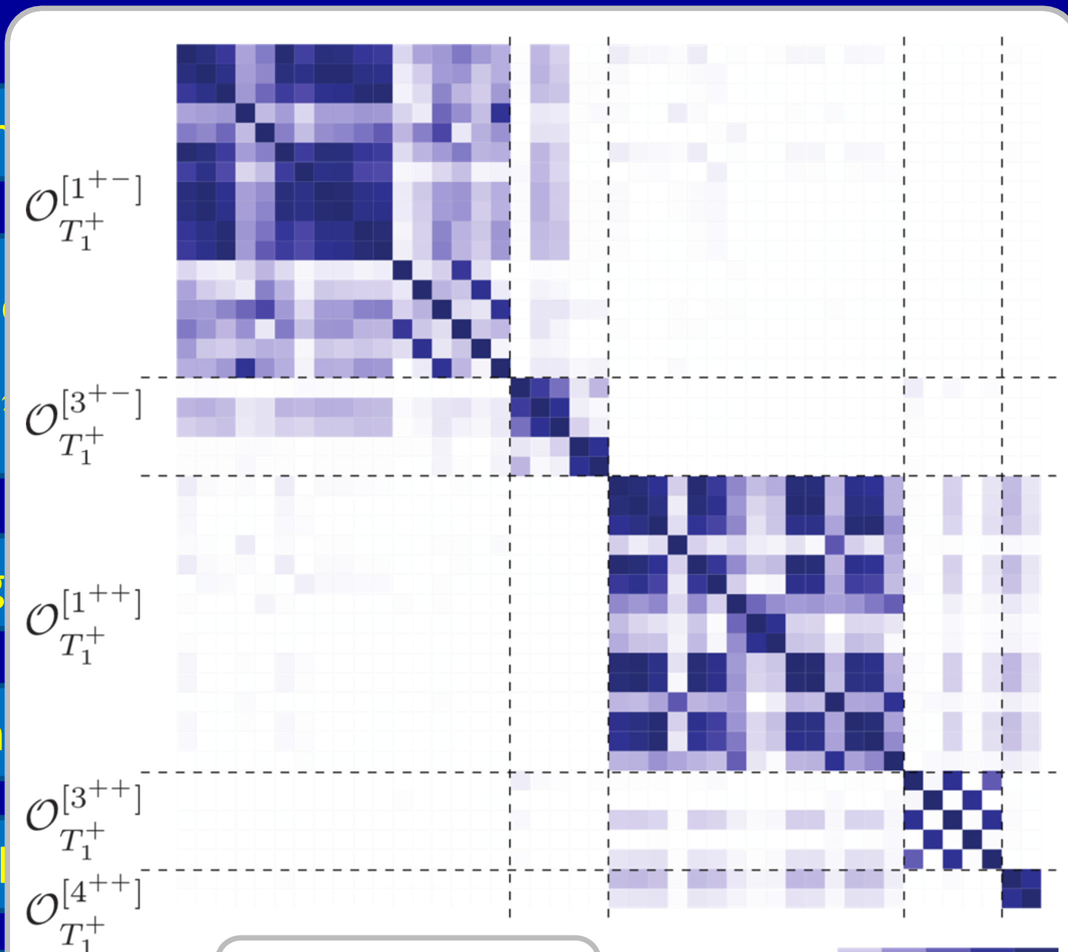
Physically axial kaon [$K_1(1270)$, $K_1(1400)$] mixing angle suggested $\approx 45^\circ$

But...

Kaons

Lower th

M_π / M
M_K / M



$$C_{ij}/\sqrt{C_{ii}C_{jj}}$$

16^3
$M_\pi \approx 400$ MeV
$M_K/M_\pi \approx 1.4$

c.f. physical
$M_K/M_\pi = 3.5$

No long … antum number

Combin

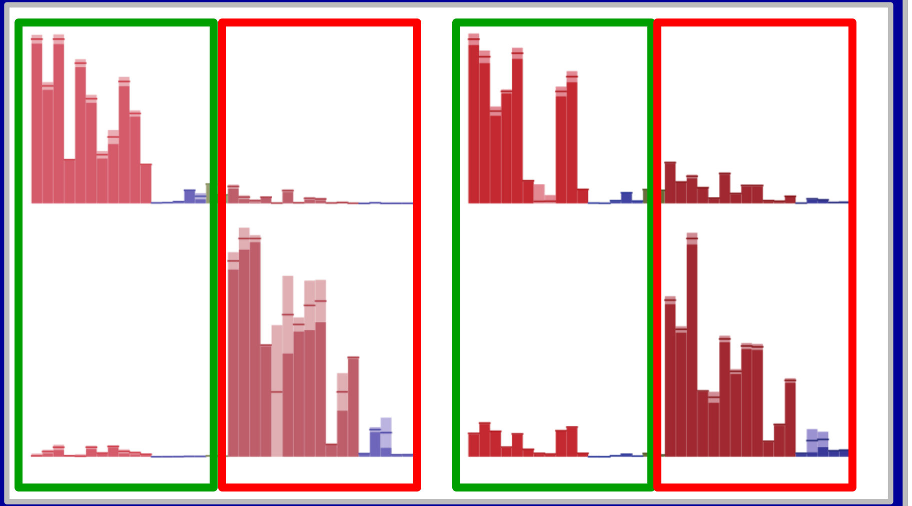
Physical … ggested ≈ 45°

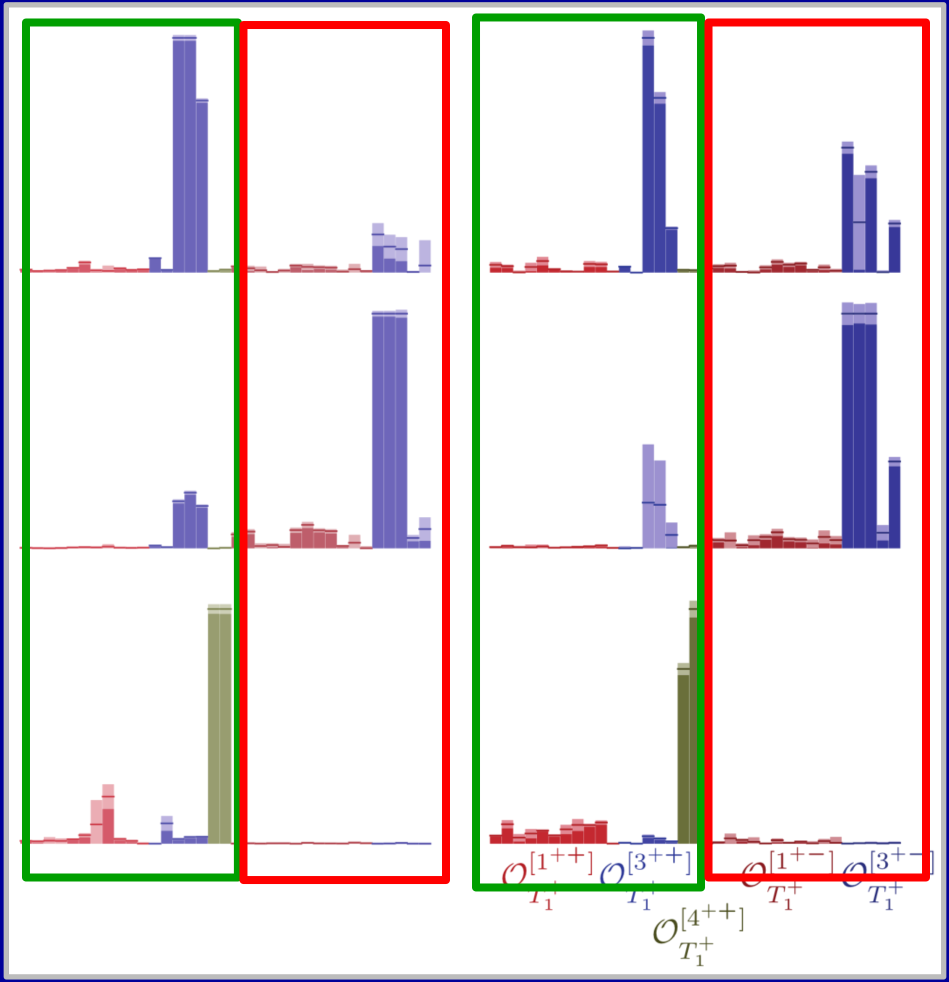
But...

Kaons – Overlaps in T_1^+

16^3
$M_\pi \approx 520$ MeV
$M_K / M_\pi \approx 1.2$

16^3
$M_\pi \approx 400$ MeV
$M_K / M_\pi \approx 1.4$

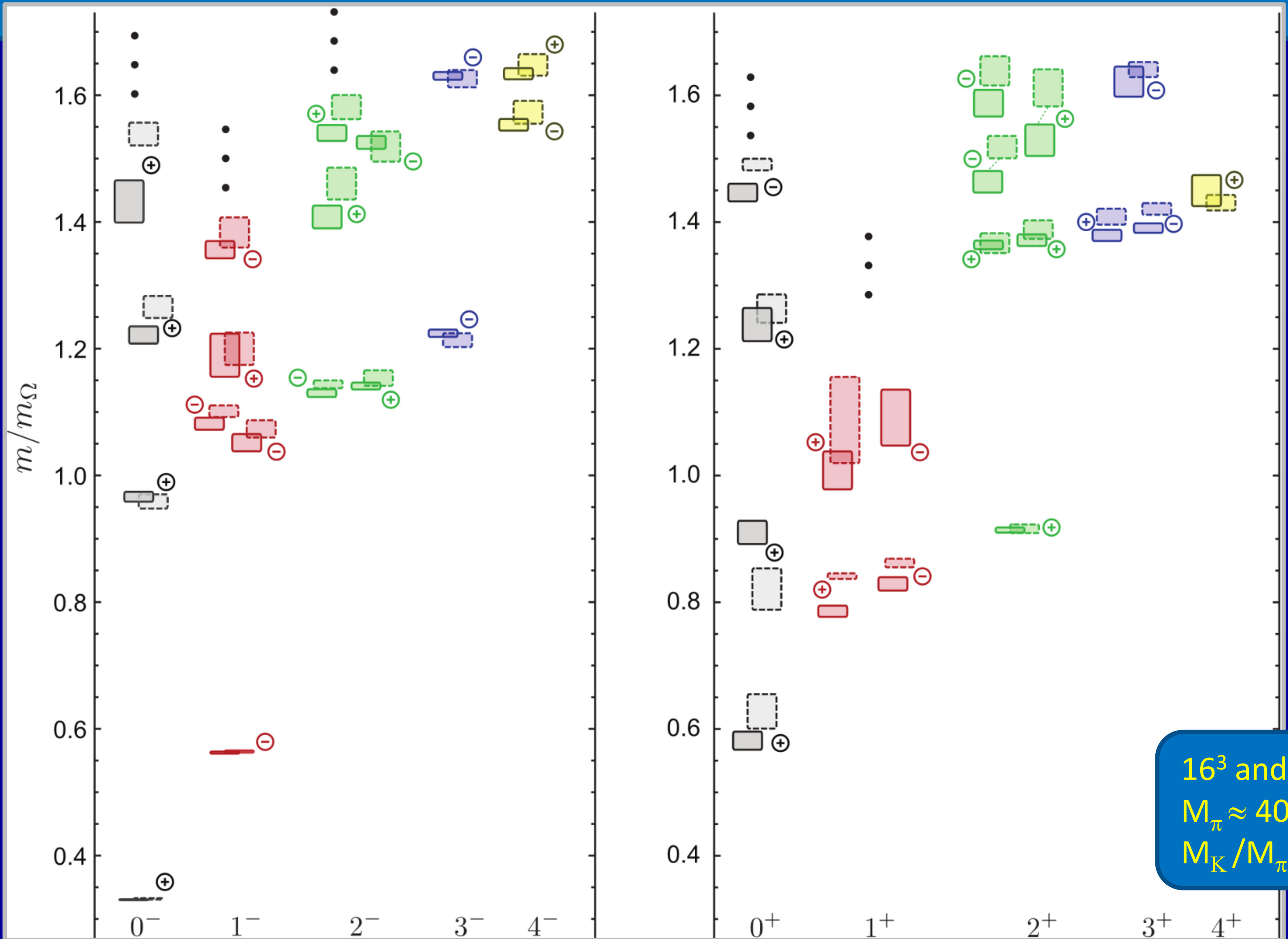




T_1^{++}

T_1^{+-}

Kaons - spectrum



16^3 and 20^3
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